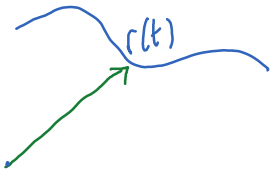
Lecture 6

Monday, May 9, 2022 10:38 AM

* Prayer

* Spiritual thought

Application of derivatives of vector functions in motion:



$r(t)$ = position at time t

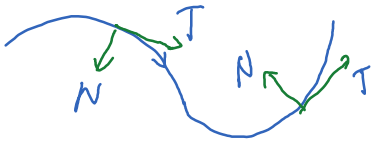
$v(t) = r'(t) = \lim_{h \to 0} \frac{r(t+h) - r(t)}{h}$ = velocity at time t

$a(t) = v'(t) = r''(t)$ = acceleration at time t

$V(t) = |v(t)| = |r'(t)|$ = speed

$a = a_T T + a_N N$: a_T is tangential acceleration

a_N is normal / centripetal acceleration



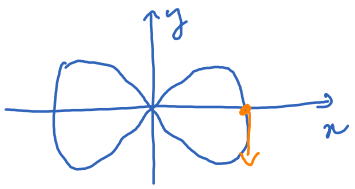
N always points toward the concave side of a curve.

$a_T = V'(t) = |r'(t)|'$

$a_N = \kappa V(t)^2 = \frac{|r' \times r''|}{|r'|^3} |r'|^2 = \frac{|r' \times r''|}{|r'|}$

Question: Is $a_T = |r''(t)|$? Answer: No.

Ex: Driving along a figure 8



$$\begin{cases} x = \sin t \\ y = \sin t \cos t \\ z = 0 \end{cases} \qquad 0 \le t \le 2\pi$$

Find the velocity / speed / acceleration components at $(1,0,0)$.

Chapter 14 : limits and derivatives of multivariable functions

$f(x,y)$ — domain, range, graph, level curves and contour map

Domain of f is $\{(x,y) \mid f(x,y) \text{ is well-defined}\}$

Range of f is the set of all values of f

Graph of f is $\{(x,y,z) \mid z = f(x,y)\}$

Level curve of f is $\{(x,y) \mid f(x,y) = \text{const}\}$

Contour map of f is a collection of many level curves of f.

Ex

$$f(x,y) = \ln(x^2 + 4y^2 - 4)$$

Domain of f is $\{(x,y) \mid x^2 + 4y^2 > 4\}$

Range of f is $\mathbb{R}$

Graph of f : use the command ICot3D in Mathematica.

Level set of f is $\{(x,y) \mid x^2 + 4y^2 = \text{const}\}$: an ellipse on the xy-plane

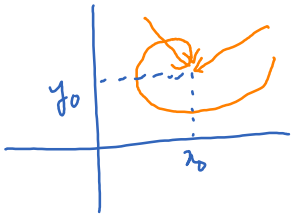
Contour map of f is a collection of non-intersecting ellipses.

Plot level curve using the command ContourPlot in Mathematica.

Ex $f(x,y,z) = x^2 - y^2 - z$

Graph 0- level set and contour map using ContourPlot3D.

* Limit of a multivariable function

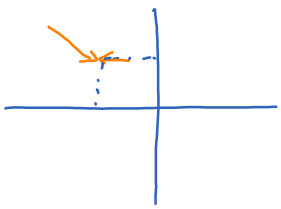


there are many ways to approach a point.

$\lim_{(x,y)\to(x_0,y_0)} f(x,y) = L$ iff $f(x,y)$ gets closer and closer to L as long as (x,y) is close enough to (x_0,y_0).

Ex $\lim_{(x,y)\to(1,0)} \frac{x}{x^2+y^2+1}$, $\lim_{(x,y)\to(1,1)} \frac{x-y}{x^2-y^2}$

Ex $\lim_{(x,y)\to(-1,1)} \frac{x+y}{x-y+2}$



Ex $\lim_{(x,y)\to(0,0)} \frac{x^3-y^3}{x^2+y^2}$

Ex

$$\lim_{(x,y)\to(0,0)} \frac{x^3}{x^2-y^2}$$